Math 104 **Midterm Exam 2 v. B** **Name:** ____________
November 8, 2019 **(Total Points: *80*)** **SID:** ____________

Information

1. **This is a closed-book test; no notes nor books are permitted.**
2. **This is a closed-phone test; no use of electronics is permitted.**
3. **Read each question carefully and answer each question completely.**
4. **Please be precise when writing your answers, and use complete sentences where appropriate (especially in proofs). You may use arithmetic manipulations freely, and you may use any result from the lectures or homework so long as:**
   - **you mention explicitly the result you are using;**
   - **the result you are using is not the same one you are being asked to prove; and**
   - **the result you are using was proved without using the result you are being asked to prove.**
5. **I will give partial credit to partial arguments; however, mere lists of facts (even true facts) that are going nowhere will receive very little or no credit. Arguments based on intuitive concepts or pictures may receive partial credit, but for full credit, a complete formal proof is required.**
6. **Write your name at the top of every page.**

Question	Points
1	15
2	15
3	20
4	15
5	15
Total:	80

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(5 points) 1. (a) Define *complete*.

(10 points) (b) Suppose that (K, d) is a compact metric space. Prove that K is complete.

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(5 points) 2. (a) Define *continuous*.

(10 points) (b) Prove that if $f : K \to X$ is continuous and K is compact, then $f(K)$ is compact.

(10 points) 3. (a) Suppose (X, d_X) and (Y, d_Y) are metric spaces, and $f : X \to Y$ is uniformly continuous. Prove that if $(a_n)_n$ is Cauchy, then $(f(a_n))_n$ is Cauchy.

(10 points) (b) Show by example that this is not true if f is merely assumed to be continuous.

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(5 points) 4. (a) Suppose $f : \mathbb{R} \to \mathbb{R}$. Define what is meant by

$$\lim_{x\to 0} f(x) = \infty.$$

(10 points) (b) Suppose that $f, g : \mathbb{R} \to \mathbb{R}$, that g is continuous with $g(x) > 1$ for all $x \in \mathbb{R}$, and that

$$\lim_{x\to 0} f(x) = \infty.$$

Prove that

$$\lim_{x\to 0} f(x)g(x) = \infty.$$

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(5 points) 5. (a) Define *equivalence relation*.

(10 points) (b) Let M be a set. Recall that metrics d, d' on M are called *strongly equivalent* if there exist $c, C > 0$ so that for all $x, y \in M$,

$$cd(x, y) \leq d'(x, y) \leq Cd(x, y).$$

Prove that strong equivalence of metrics is an equivalence relation.

Metrics d and d' are called *equivalent* or *topologically equivalent* if subsets of M are open with respect to d if and only if they are open with respect to d'. This is also an equivalence relation, and strongly equivalent metrics are always topologically equivalent.