Math 104 **Midterm Exam 1** **Name:** ______________________

October 4, 2019 **(Total Points: *60*)** **SID:** ______________________

Information

1. **This is a closed-book test; no notes nor books are permitted.**
2. **This is a closed-phone test; no use of electronics is permitted.**
3. **Read each question carefully and answer each question completely.**
4. **Please be precise when writing your answers, and use complete sentences where appropriate (especially in proofs). You may use arithmetic manipulations freely, and you may use any result from the lectures or homework so long as:**
   - **you mention explicitly the result you are using;**
   - **the result you are using is not the same one you are being asked to prove; and**
   - **the result you are using was proved without using the result you are being asked to prove.**
5. **I will give partial credit to partial arguments; however, mere lists of facts (even true facts) that are going nowhere will receive very little or no credit. Arguments based on intuitive concepts or pictures may receive partial credit, but for full credit, a complete formal proof is required.**
6. **Write your name at the top of every page.**

Question	Points
1	15
2	15
3	15
4	15
Total:	60

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(5 points) 1. (a) Define what it means for a set to be *open*.

(10 points) (b) Prove that if G is an open subset of a metric space (M, d), then G^c is closed.

(5 points) 2. (a) Define *lower bound* and *infimum/greatest lower bound*.

(10 points) (b) Let $S = \{x + y \mid x, y \in \mathbb{R}, 1 < x, 2x < y\}$. Determine if $\inf S$ exists in $\mathbb{R}$, and if it does, find its value. Remember to supply a proof.

(5 points) 3. (a) Define what it means for a set to be *compact*.

(10 points) (b) Let $K = \left\{-n^2 \mid n \in \mathbb{N}\right\} \subseteq \mathbb{Z}$, with the usual metric:

$$\begin{aligned} d : \mathbb{Z} \times \mathbb{Z} &\to \mathbb{R}_{\geq 0} \\ (j, k) &\mapsto |k - j| \,. \end{aligned}$$

Determine, with proof, if K is compact.

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(5 points) 4. (a) Define what it means for a sequence to be *convergent* in a metric space (M, d).

(10 points) (b) Suppose that $(a_n)_n$ is a sequence in a metric space (M, d), which converges to a limit $a \in M$. Prove that $K = \{a_n \mid n \in \mathbb{N}\} \cup \{a\}$ is compact.