
Information

1. This is a closed-book test; no notes nor books are permitted.
 2. This is a closed-phone test; no use of electronics is permitted.
 3. Read each question carefully and answer each question completely.
 4. Please be precise when writing your answers, and use complete sentences where appropriate (especially in proofs). You may use arithmetic manipulations freely, and you may use any result from the lectures or home-work so long as:
 - you mention explicitly the result you are using;
 - the result you are using is not the same one you are being asked to prove; and
 - the result you are using was proved without using the result you are being asked to prove.
 5. I will give partial credit to partial arguments; however, mere lists of facts (even true facts) that are going nowhere will receive very little or no credit. Arguments based on intuitive concepts or pictures may receive partial credit, but for full credit, a complete formal proof is required.
 6. Write your name at the top of **every** page.
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Question	Points
1	15
2	15
3	15
4	15
Total:	60

(5 points) 1. (a) Define what it means for a sequence $(x_k)_k$ in a metric space (M, d) to be *Cauchy*.

(10 points) (b) Suppose that $(x_k)_k$ is a Cauchy sequence in a metric space (M, d) . Prove that $(x_k)_k$ converges if it has a convergent subsequence.

(5 points) 2. (a) Define *continuous*.

(10 points) (b) Suppose that (A, d_A) and (B, d_B) are metric spaces, and $g : A \rightarrow B$ is continuous. Prove that for every open subset $U \subseteq B$, the set $g^{-1}(U) = \{a \in A \mid g(a) \in U\}$ is open.

(5 points) 3. (a) Define *uniformly continuous*.

(10 points) (b) Suppose $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are uniformly continuous. Prove that their composition $g \circ f : X \rightarrow Z$ is uniformly continuous. (Recall that $g \circ f(x) = g(f(x))$.)

(5 points) 4. (a) Suppose $f : [0, \infty) \rightarrow \mathbb{R}$, and $L \in \mathbb{R}$. State precisely what is meant by

$$\lim_{t \rightarrow \infty} f(t) = L.$$

(10 points) (b) Suppose $f : [0, \infty) \rightarrow \mathbb{R}$ is continuous, and that $\lim_{t \rightarrow \infty} f(t)$ converges (to an element of $\mathbb{R}$).
Prove that f is bounded.
(That is, show that the set $f([0, \infty)) = \{f(t) \mid t \in [0, \infty)\}$ is bounded.)